The small-scale magnetic field and the evolution of pulsar rotation in the framework of three-component model of neutron star

M. V. Vorontsov[1], **K. Yu. Kraav**[1], **D. P. Barsukov**[1,2], **O. A. Goglichidze**[2]

[1] Peter the Great St.Petersburg Polytechnic University [2] Ioffe Institute

Abstract

We consider the evolution of pulsar rotation assuming that the star consists of crust component (which rotation is observed) and two core components. All components are supposed to rotate as rigid bodies. One of the core components contains pinned superfluid which can, for some reasons, suddenly inject some small fraction of stored angular momentum in it. In the framework of this toy model the star can demonstrate glitch-like events together with long period precession (with period $10 - 10^4$ years). This precession can survive on pulsar braking time-scale if the external electromagnetic torque acting on neutron star has an equilibrium inclination angle. It can take place for instance if pulsar tubes are bended by small-scale magnetic field.

1. three-component model

Let us assume that a neutron star consists of three components [1]:

1. **c-component** is outer component including NS crust. It rotates with angular velocity $\vec{\Omega}$:

$$\vec{M}_c = I_c\vec{\Omega}, \quad \dot{\vec{M}}_c = \vec{K}_{ext} + \vec{N}_c,$$

where $\vec{M}_c$ is the crust angular momentum, I_c is its moment of inertia, $\vec{K}_{ext}$ is external torque acting on the crust, $\vec{N}_c$ is torque due to interaction with other componets.

g-component is inner component rotating with angular velocity $\vec{\Omega}_g$:

$$\vec{M}_g = I_g\vec{\Omega}_g + \vec{L}_g, \quad \dot{\vec{M}}_g = \vec{N}_g, \quad \dot{\vec{L}}_g = \left[\vec{\Omega}_g \times \vec{L}_g\right],$$

where $\vec{M}_g$ is the total angular momentum of g-component, $I_g\vec{\Omega}_g$ is the angular momentum of normal matter, I_g is the moment of inertia of normal matter, $\vec{L}_g$ is the angular momentum of pinned superfluid, $\vec{N}_g$ is the torque due to interaction with other componets.

2. **r-component** is inner component rotating with angular velocity $\vec{\Omega}_r$:

$$\vec{M}_r = I_r\vec{\Omega}_r, \quad \dot{\vec{M}}_r = \vec{N}_r,$$

where $\vec{M}_r$ is the angular momentum of r-component, I_r is its moment of inertia, $\vec{N}_r$ is torque due to interaction with other components.

2. Interaction between the components

Let us assume that

$$\vec{N}_c = \vec{N}_{rc} + \vec{N}_{gc}, \ \vec{N}_g = \vec{N}_{cg} + \vec{N}_{rg}, \ \vec{N}_r = \vec{N}_{cr} + \vec{N}_{gr},$$
$$\vec{N}_{ij} = -I_j\left(\alpha_{ij}\mu^{||}_{ij} + \beta_{ij}\mu^{\perp}_{ij} + \gamma_{ij}[\vec{e}_\Omega \times \mu^{\perp}_{ij}]\right),$$

where $\alpha_{ij}, \beta_{ij}, \gamma_{ij} = const$, $\vec{\mu}_{ij} = \vec{\Omega}_j - \vec{\Omega}_i$, $i, j = (c, g, r)$, $A^{||} = (\vec{e}_\Omega \cdot \vec{A})$, $\vec{A}^{\perp} = \vec{A} - \vec{e}_\Omega(\vec{e}_\Omega \cdot \vec{A})$, $\vec{e}_\Omega = \vec{\Omega}/\Omega$.

Further we will assume that

$$\beta_{cg} = \alpha_{cg} \text{ and } \gamma_{cg} = 0, \ \beta_{gr} = \beta_{cr} = \Omega\frac{\sigma}{1+\sigma^2}, \ \alpha_{gr} = \alpha_{cr} = 2\beta_{cr}, \ \gamma_{gr} = \gamma_{cr} = -\sigma\,\beta_{cr}$$

$\Omega\sigma \ll \alpha_{cg} \ll \Omega$

3. Quasistatic regime

Let us assume that the torque $\vec{K}_{ext}$ slowly varies with time in the crust frame of reference. In the absence of glitch-like events the system evolves to the quasistatic rotation regime which described by equations

$$\dot{\Omega} = \frac{K^{||}_{ext}}{I_{tot}}, \qquad \Omega\dot{\vec{e}}_\Omega = \mathrm{B}\frac{\vec{K}^{\perp}_{ext}}{I_c} + \Gamma\,[\vec{e}_\Omega \times \frac{\vec{K}^{\perp}_{ext}}{I_c}],$$

where $\mathrm{B} + i\Gamma = \frac{\Omega}{\Delta}\left(\Omega - i\left(z_{gr} + z_{cr}\right)\right), \quad z_{ij} = \beta_{ij} + i\,\gamma_{ij}, \quad I_{tot} = I_c + I_g + I_r$

$\Delta = \Omega^2 - i\,\Omega\left(z_{gr} + z_{cr} + z_{gc} + z_{rc}\right) - \left(z_{gr}\,z_{gc} + z_{gr}\,z_{rc} + z_{cr}\,z_{gc}\right)$

In the weak viscosity limiting case ($|z_{ij}| \ll \Omega$)

$$\mathrm{B} \approx 1 - \frac{\gamma_{gc} + \gamma_{cr}}{\Omega}, \qquad \Gamma \approx \frac{\beta_{gc} + \beta_{cr}}{\Omega}$$

$\vec{\mu}^{\perp}_{cg} \approx \vec{\mu}^{\perp}_{cr} \approx \frac{1}{\Omega}\left[\vec{e}_\Omega \times \frac{\vec{K}^{\perp}_{ext}}{I_c}\right]$ and $\vec{L}^{\perp}_g \approx \frac{L^{||}_g}{\Omega^2}\left[\vec{e}_\Omega \times \frac{\vec{K}^{\perp}_{ext}}{I_c}\right]$

4. glitch-like event

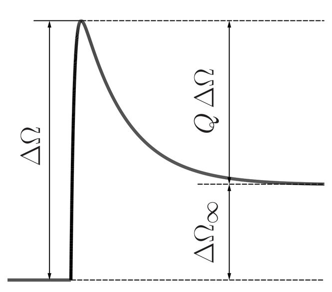
Let us assume that the pinned superfluid injects a small fraction $I_g\Delta\mu$ of stored angular momentum L_g in g-component at $t = 0$:

$\mu^{||}_{cr}\Big|_{t=0} = 0$ and $\mu^{||}_{cg}\Big|_{t=0} = \Delta\mu$

$$\Omega(t) = \Omega|_{t=0} + \Delta\Omega\left(1 - e^{-p_+ t} - Q(1 - e^{-p_- t})\right)$$

$\Delta\Omega = \frac{\Delta\Omega_\infty}{1-Q}$ and $\Delta\Omega_\infty = \frac{I_g\Delta\mu}{I_{tot}}$

$$Q = \frac{I_{tot}\alpha_{cg} - I_c p_+}{I_{tot}\alpha_{cg} - I_c p_-}$$



where p_+, p_- are the roots of equation

$$p^2 - \left(\alpha_{gc} + \alpha_{cr} + \alpha_{gr} + \alpha_{cr} + \alpha_{cg} + \tilde{\alpha}_{rg}\right)p +$$
$$+ \left(\alpha_{gc} + \alpha_{cg} + \tilde{\alpha}_{rg}\right)\left(\alpha_{cr} + \alpha_{gr} + \alpha_{cr}\right) - \left(\tilde{\alpha}_{rg} - \alpha_{cr}\right)\left(\alpha_{gr} - \alpha_{gc}\right) = 0$$

If one assumes that

$$\alpha_{cg} \gg \left(1 + \frac{I_c}{I_r}\right)\alpha_{cr}, \ \left(1 + \frac{I_g}{I_r}\right)\tilde{\alpha}_{rg}$$

than

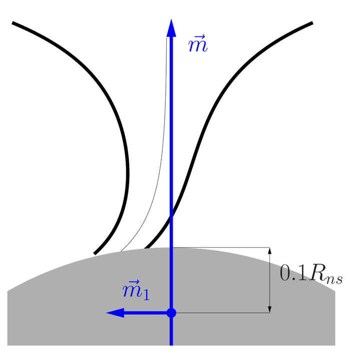
$$p_+ \approx \left(1 + \frac{I_c}{I_g}\right)\alpha_{cg} \quad p_- \approx \frac{I_{tot}}{I_c + I_g}\left(\alpha_{gr} + \alpha_{cr}\right) \quad \text{and} \quad Q \approx \frac{I_r}{I_{tot}}$$

$1/p_+ \lesssim 30$ s [2, 3]
$1/p_- \sim 1 - 30$ days [4]

5. Small-scale magnetic field and the external torque

The details of the model describing the influence of small-scale field on external torque see in [9].

1. Pulsar is braked by both magneto-dipolar and current mechanisms
2. Pulsar tubes are bended by small-scale magnetic field
3. Inner gaps
4. Steady charge limited flow
5. Gaps are placed as low as possible
6. The currents through the both gaps are the same



$$\vec{K}_{ext} = -K_0\left(\vec{e}_\Omega - \vec{e}_m(1-\alpha)\cos\chi - R_{eff}[\vec{e}_\Omega \times \vec{e}_m]\cos\chi\right)$$

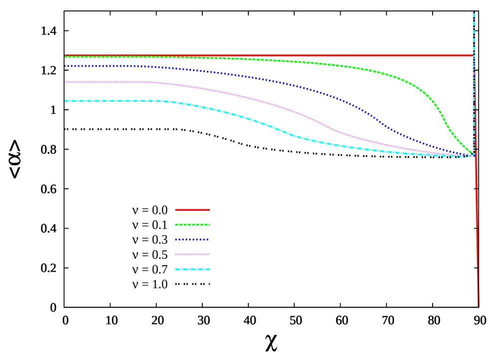
$$K_0 = \frac{2}{3}\frac{m^2}{c^3}\Omega^3, \quad R_{eff} = \frac{3}{2}\left(\frac{c}{\Omega r_{ns}}\right)\delta \sim 5\cdot 10^3\left(\frac{P}{1\,\text{s}}\right), \quad \delta = \frac{3}{5}\text{[5]}$$

$\alpha(\chi, \phi_\Omega) = \frac{3}{4}\frac{j_N + j_S}{j_{GJ}}$, where

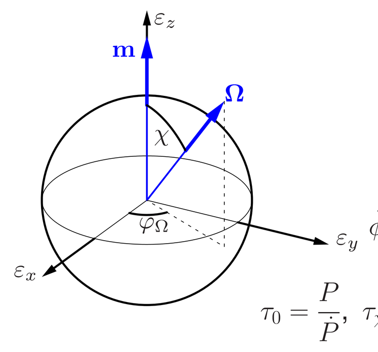
$$j_{GJ} = \frac{\Omega B_{dip}}{2\pi c}\cos\chi,$$

It is more convenient to use function α averaged over the precession angle:

$$<\alpha>(\chi) = \frac{1}{2\pi}\int_0^{2\pi}\alpha(\chi, \phi_\Omega)d\phi_\Omega$$



6. Equations of pulsar rotation



$$\dot{\Omega} = -\frac{\Omega}{\tau_0}\left(\sin^2\chi + \alpha\cos^2\chi\right)$$
$$\dot{\chi} = -\frac{I_{tot}}{I_c}\frac{1}{\tau_0}\left(\mathrm{B}\left(1-\alpha\right) - \Gamma\,R_{eff}\right)\sin\chi\,\cos\chi$$
$$\dot{\phi}_\Omega = -\frac{I_{tot}}{I_c}\frac{1}{\tau_0}\left(\mathrm{B}\,R_{eff} + \Gamma\left(1-\alpha\right)\right)\cos\chi$$

$$\tau_0 = \frac{P}{\dot{P}}, \ \tau_\chi \sim \frac{I_c}{I_{tot}}\tau_0, \ T_p \approx \frac{I_c}{I_{tot}}\frac{2\pi\tau_0}{\mathrm{B}R_{eff}}$$

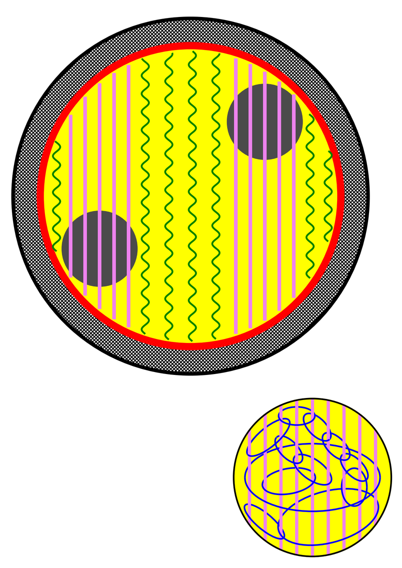
Here τ_0 is the pulsar braking time-scale, τ_χ is time-scale of inclination angle evolution, T_p is the period of precession.

$$T_p \ll \tau_\chi \ll \tau_0$$

One can average the equations over precession period:

$$\frac{d\chi}{dP} = -\frac{1}{P}\cdot\frac{I_{tot}}{I_c}\cdot\frac{\mathrm{B}\left(1 - <\alpha>\right) - \Gamma\,R_{eff}}{\sin^2\chi + <\alpha>\cos^2\chi}\cdot\sin\chi\cos\chi$$

7. Large g-component model



1. **c-component** consists of the crust and outer non-superconductive layer of the core, $I_c \sim 10^{-2}I_{tot}$.
2. **g-component** consists of freely moving bundles of fluxoids and normal matter inside the bundles, $I_g \sim 10^{-3}I_{tot}$, $L_g \sim 10^{-2}I_{tot}\Omega$. Small magnetic flux may escape from the bundles providing strong interaction with the crust.
3. **r-component** consists of normal and superfluid matter of the core, $I_r \approx I_{tot}$. The interaction with the c-component is provided by the normal matter viscosity. The interact with g-component is provided by the friction between normal matter and vortices interaction.

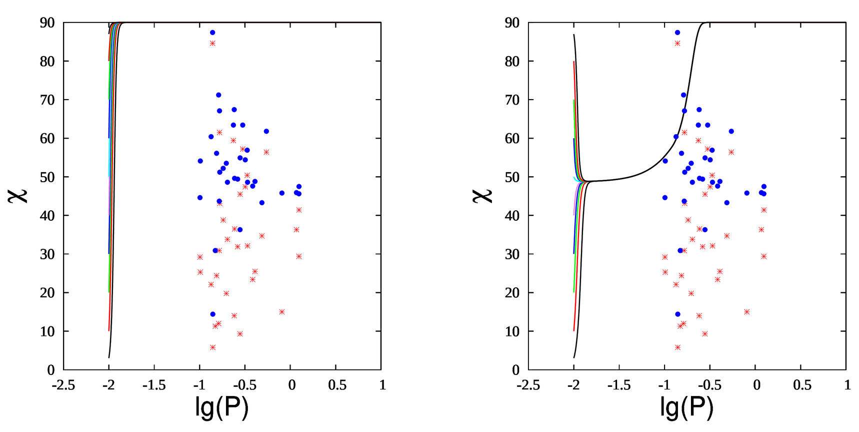


Fig.1 The evolution of inclination angle for different initial values of χ, $\nu = 0.0$ (left panel) and $\nu = 0.5$ (right panel), $I_c/I_{tot} = 10^{-2}$, $I_g/I_{tot} = 10^{-3}$, $\alpha_{cg} = 10^{-1}$ s^{-1}, $\sigma = 10^{-10}$. Observed inclination angles β_2 at 10 cm is taken from [7], red stars correspond to $C > 0$, blue dots correspond to $C < 0$.

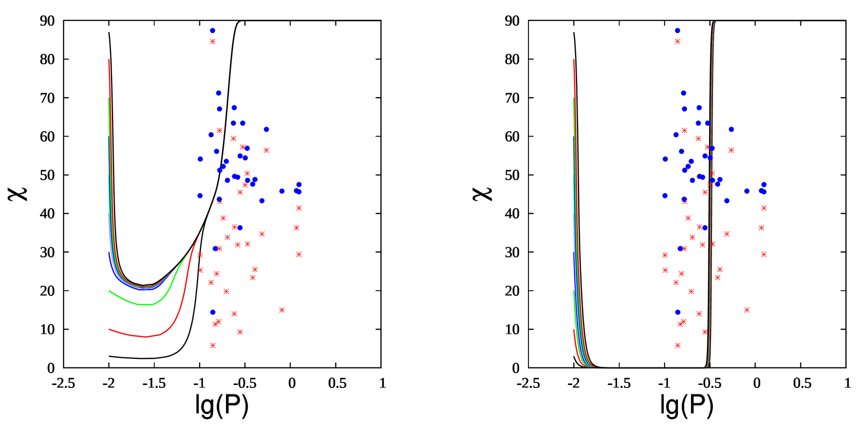


Fig.2 The same as in fig.1 but for $\nu = 0.8$ (left panel) and $\nu = 1.0$ (right panel).

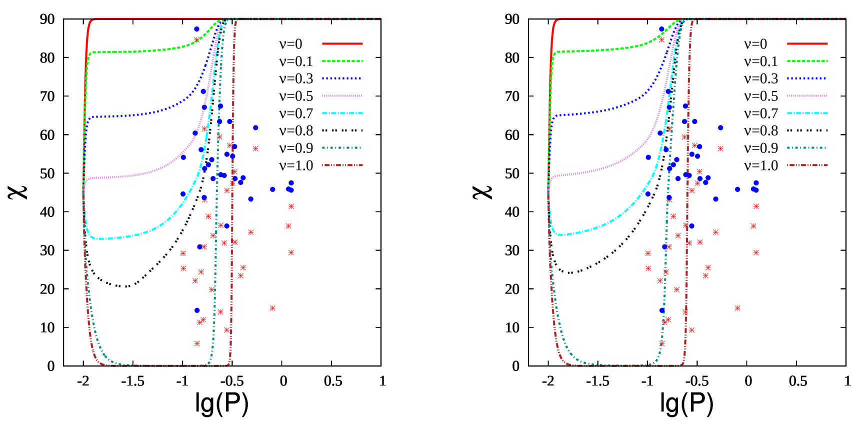


Fig.3 The evolution of inclination angle χ for different values of ν, $\sigma = 10^{-10}$ (left panel), $\sigma = 10^{-6}$ (right panel), $I_c/I_{tot} = 10^{-2}$, $I_g/I_{tot} = 10^{-3}$, $\alpha_{cg} = 10^{-1}$ s^{-1}. Initial angle $\chi = 45°$.

8. Small g-component model

Let us assume that the NS possesses a rigid inner core with radius r_g = 0.1 r_{ns}

1. **c-component** ($I_c \sim 0.1I_{tot}$) consists of the crust and the core normal matter
2. **g-component** ($I_g \sim \left(\frac{r_g}{r_{ns}}\right)^5 I_{tot} \sim 10^{-5}I_{tot}$, $L_g \sim \left(\frac{r_g}{r_{ns}}\right)^2 I_s\Omega \sim 10^{-2}I_{tot}\Omega$) consists of normal matter of "rigid core" and vortices pinned to it
3. **r-component** ($I_r \approx I_{tot}$) consists of superfluid neutrons of the NS core

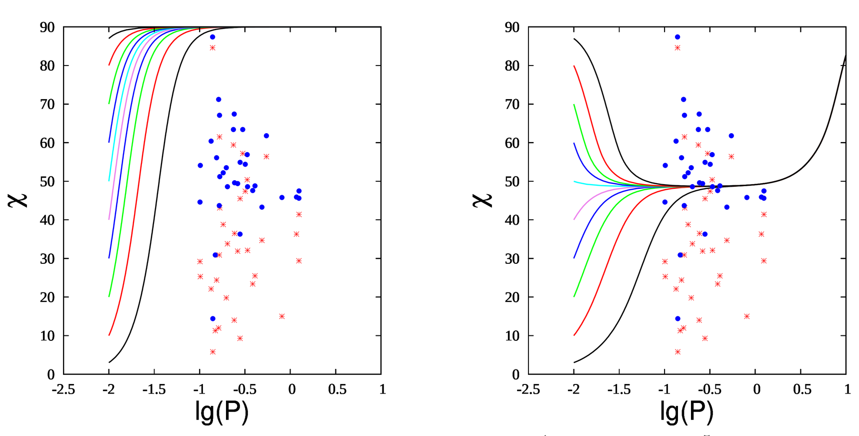


Fig.4 The same as in fig.1 but for $I_c/I_{tot} = 10^{-1}$, $I_g/I_{tot} = 10^{-5}$.

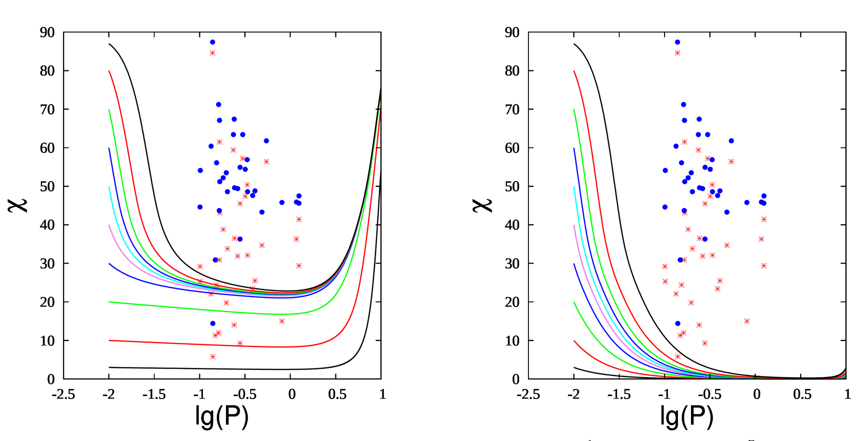


Fig.5 The same as in fig.2 but for $I_c/I_{tot} = 10^{-1}$, $I_g/I_{tot} = 10^{-5}$.

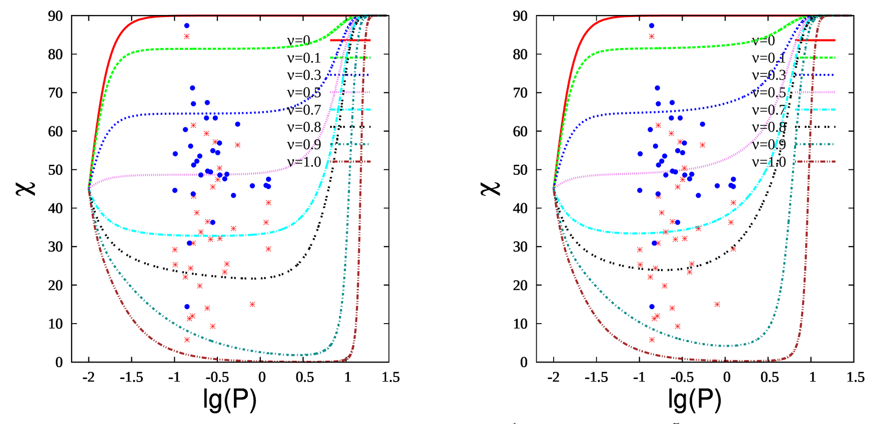


Fig.6 The same as in fig.3 but $I_c/I_{tot} = 10^{-1}$, $I_g/I_{tot} = 10^{-5}$.

9. Triaxial precession

$$\dot{\vec{M}}_c = \vec{K}_{ext} + \vec{N}_c \text{ and } \vec{M}_c = I_c\left(\vec{\Omega} + \sum_\alpha\varepsilon_\alpha\vec{e}_\alpha(\vec{e}_\alpha\cdot\vec{\Omega})\right)$$
$$\vec{\xi} = \sum_\alpha\varepsilon_\alpha\left(\vec{e}_\Omega\right)_\alpha\vec{e}_\alpha, \ \zeta = \sum_\alpha\varepsilon_\alpha\left(\vec{e}_\Omega\right)^2_\alpha \text{ and } \varepsilon_\alpha = \frac{I_{c\alpha} - I_c}{I_c}$$

In quasistatic approximation

$\vec{N}_c = -(I_g + I_r)\,\dot{\Omega}\,\vec{e}_\Omega - I_c\Omega\left(\tilde{\Gamma}\dot{\vec{e}}_\Omega - \tilde{B}[\vec{e}_\Omega \times \dot{\vec{e}}_\Omega]\right)$ where $\tilde{B} + i\tilde{\Gamma} = \frac{1}{\Omega}\left(z_{gc} + z_{rc} + \frac{i\,z_{rc}z_{cr}}{\Omega - i(z_{gr} + z_{cr})}\right)$

$$\dot{\Omega} = \frac{K^{||}_{ext}}{I_{tot}}, \qquad \dot{\zeta} = \frac{2\tilde{B}}{(1+\tilde{\Gamma})^2 + \tilde{B}^2}\cdot\Omega\cdot(\xi^2 - \zeta^2) + \frac{2}{I_c\Omega}\cdot\frac{(1+\tilde{\Gamma})(\vec{\xi}\cdot\vec{K}^{\perp}_{ext}) + \tilde{B}(\vec{\xi}\cdot[\vec{e}_\Omega\cdot\vec{K}^{\perp}_{ext}])}{(1+\tilde{\Gamma})^2 + \tilde{B}^2}$$

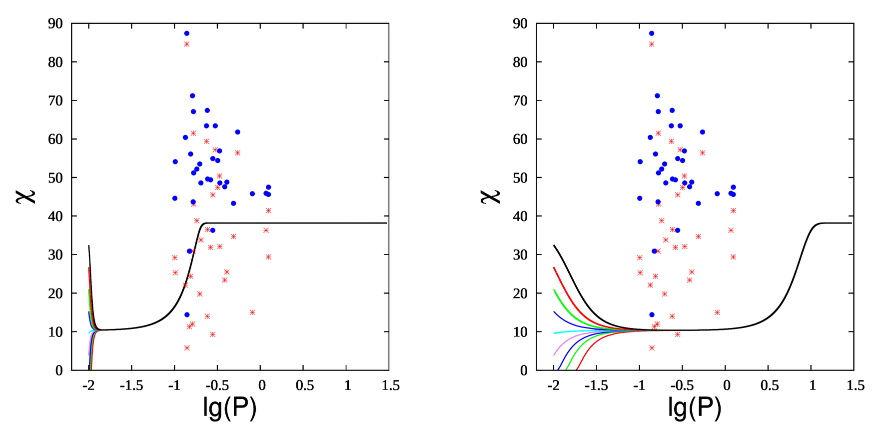


Fig.7 The same as in fig.1 but for triaxial star with $\varepsilon_z = 1$, $\varepsilon_y = 0$, $\varepsilon_x = -0.5$, $\vec{m} = \vec{e}_y\sin\Psi_0 + \vec{e}_z\cos\Psi_0$, $\Psi_0 = 10°$, $\vec{m}_1 = \vec{e}_x$, $\alpha_{cg} = 10^{-1}$ s^{-1}, $\sigma = 10^{-10}$, $\nu = 0.5$, $I_c/I_{tot} = 10^{-1}$, $I_g/I_{tot} = 10^{-3}$ (left panel), $I_c/I_{tot} = 10^{-2}$, $I_g/I_{tot} = 10^{-5}$ (right panel).

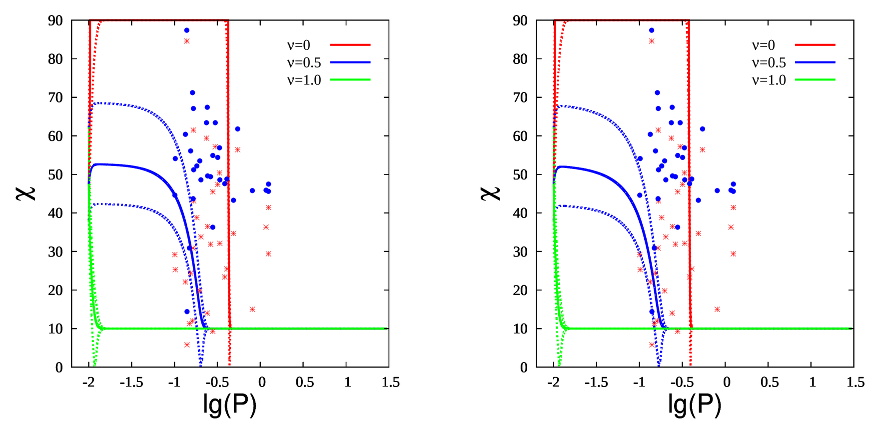


Fig.8 The same as in fig.3 but for triaxial star with $\varepsilon_z = 1$, $\varepsilon_y = 0$, $\varepsilon_x = -0.5$, $I_c/I_{tot} = 10^{-2}$, $I_g/I_{tot} = 10^{-3}$, and $\alpha_{cg} = 10^{-1}$ s^{-1}, $\sigma = 10^{-10}$ (left panel), $\sigma = 10^{-6}$ (right panel). Average value of angle χ is shown by solid line, its maximum and minimum values is shown by dashed lines.

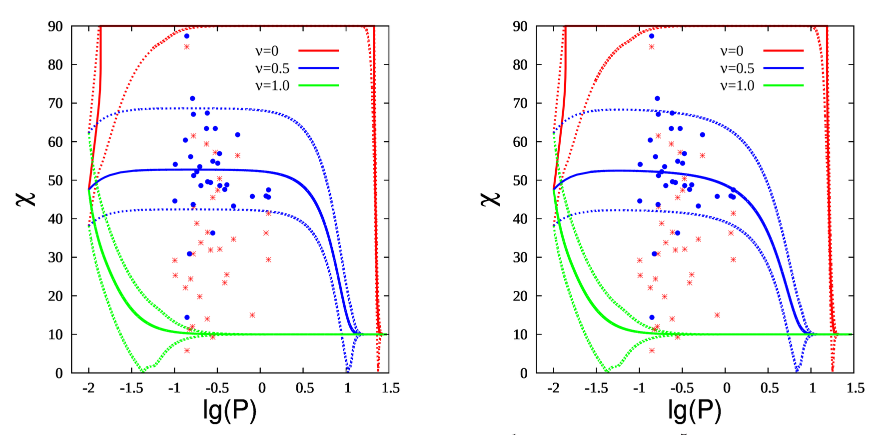


Fig.9 The same as in fig.8 but $I_c/I_{tot} = 10^{-1}$, $I_g/I_{tot} = 10^{-5}$.

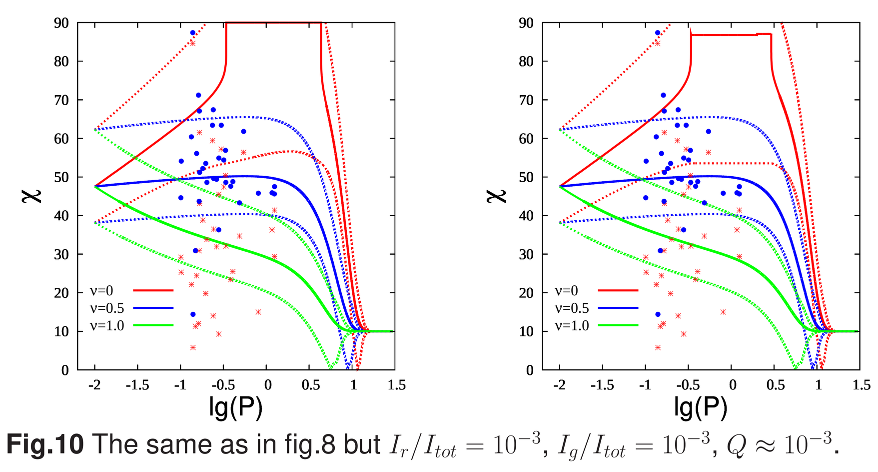


Fig.10 The same as in fig.8 but $I_r/I_{tot} = 10^{-3}$, $I_g/I_{tot} = 10^{-3}$, $Q \approx 10^{-3}$.

10. Conclusions

It is shown that the small g-component seems be in better agreement with observations.

Problems

1. What is the nature of g-component and how large is its lifetime?
2. Is this possible that a small amount of normal matter rules the large amount of superfluid ($I_g\Omega \sim 10^{-3}L_g$)?
3. Relaxation time is always the same.

Possible solutions

1. The friction between c-component and g-component increases during glitch. Angular momentum is transfered from g-component to c-component by Kelvin waves [8], Alfven or sound waves.
2. The g-component collapses on a time-scale of the pulsar lifetime

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References

[1] A.Sedrakian, I.Wasserman, J.M.Cordes // ApJ V. **524**, p.341 (1999)
[2] R.G. Dodson, P.M. McCulloch, D.R. Lewis // ApJ, V. **564**, p. L85 (2002)
[3] B.Haskell, A.Melatos // International Journal of Modern Physics D, V. **24**, id. 1530008 (2015)
[4] F. D'Alessandro // Astrophysics and Space Science, V. **246**, p.73 (1996)
[5] A.Melatos // MNRAS, V. **313**, p.217 (2000)
[6] K.Glampedakis, P.D.Lasky // MNRAS V. **450**, p.1638 (2015)
[7] I.F.Malov, E.B.Nikitina // Astronomy Reports, V. **55**, p. 19 (2011)
[8] B.Link // ApJ V. **789**, id 141 (2014)
[9] D.P. Barsukov, P.I. Polyakova, A.I. Tsygan // Astronomy Reports, V. **53**, p. 1146 (2009)