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Dynamics of the Flows Accreting onto a Magnetized Neutron Star

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Abstract—We investigate the unsteady column accretion of material at a rate $10^{15}\,\mathrm{g\,s^{-1}} \leq \dot{M} \leq 10^{16}\,\mathrm{g\,s^{-1}}$ onto the surface of a magnetized neutron star using a modified first-order Godunov method with splitting. We study the dynamics of the formation and evolution of a shock in an accretion column near the surface of a star with a magnetic field $5 \times 10^{11} \leq B \leq 10^{13}$ G. An effective transformation of the accretion flow energy into cyclotron radiation is shown to be possible for unsteady accretion with a collisionless shock whose front executes damped oscillations. The collisionless deceleration of the accreting material admits the conservation of a fraction of the heavy nuclei that have not been destroyed in spallation reactions. The fraction of the CNO nuclei that reach the stellar atmosphere is shown to depend on the magnetic field strength of the star. © *2004 MAIK "Nauka/Interperiodica"*.

Key words: *plasma astrophysics, hydrodynamics and shock waves, accretion, neutron stars.*

INTRODUCTION

The accretion onto compact objects has been considered as an effective source of hard X-ray radiation for about forty years (see Zel'dovich 1964; Salpeter 1964; Shakura and Sunyaev 1973).

Several analytical, semianalytical, and numerical models of accretion onto neutron stars (NSs) and black holes have been constructed (see, e.g., Frank *et al.* 2002). However, some of the important questions formulated even in pioneering papers have no clear answers as yet. One of the fundamental questions is the nature of the accreting material: Is it a gas of interacting particles (hydrodynamic regime) or a collection of separate noninteracting particles (free-fall regime)? Zel'dovich (1967) and Zel'dovich and Shakura (1969) considered these two regimes and showed that the radiation spectrum near the stellar surface depends significantly on the accretion regime under consideration.

Up to now, the authors of the models have provided circumstantial qualitative evidence to substantiate a particular regime in the hope that a realistic model supported by experimental evidence can *a posteriori* justify the choice of a regime, at least for a certain domain of parameters of the accretion system. Under which conditions collisionless shocks that decelerate the material as it moves toward the stellar surface arise in an accretion flow is another related question.

Bisnovatyi-Kogan and Fridman (1969) pointed out that a *collisionless* shock could appear in the flow accreting onto a NS if the star possesses a dipole magnetic field of $B \sim 10^8$ G.

The shock that decelerates the flow accreting onto a NS in a binary system plays a key role in the model by Davidson and Ostriker (1973). In their models, Shapiro and Salpeter (1975), Basko and Sunyaev (1976), Langer and Rappoport (1982), and Braun and Yahel (1984) considered the accretion onto a NS under various assumptions about the magnetic field strength of the star and found *steady-state* solutions of the system of hydrodynamic equations for the accretion flow. The models by Shapiro and Salpeter (1975) and Langer and Rappoport (1982) postulate the existence of a *stationary collisionless* shock at a certain height above the surface, which is a parameter of these models. Basko and Sunyaev (1976) demonstrated the accretion regime with a *radiative* shock in the NS atmosphere. In their model, Braun and Yahel (1984) showed that a *stationary collisionless* shock could exist above the surface of a magnetized NS only when the accretion rate is low enough (more specifically, does not exceed a few percent of the Eddington value).

Detailed models for the two-dimensional unsteady *radiation-dominated super-Eddington* accretion onto a magnetized NS have been developed by Arons, Klein, and coauthors (see Klein and Arons 1989; Klein *et al.* 1996; and references therein). The authors of these models show that, if the accretion rate is high enough (more specifically, if the accretion is essentially super-Eddington one), then *nonstationary radiation-dominated* shocks appear and evolve in the column. An important peculiarity of these studies

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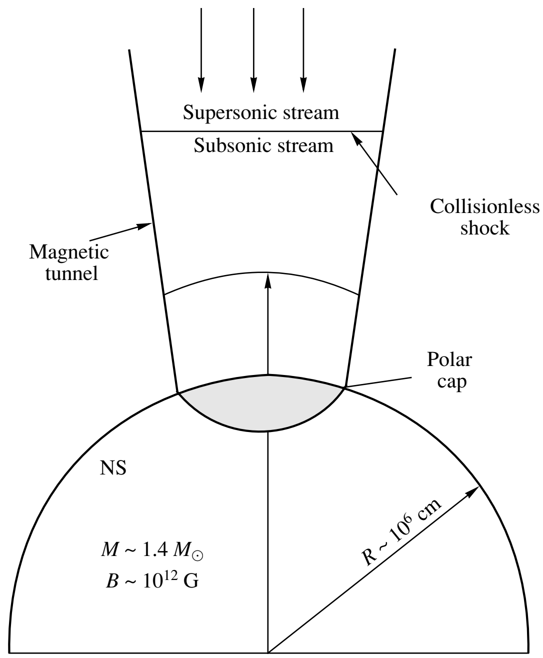

Fig. 1. Geometry of the accretion column.

is that the existence of a shock in the column is not postulated, but is the result of model evolution of the system.

In this paper, we present a numerical model for the *sub-Eddington* one-dimensional unsteady hydrodynamic (in the above sense) accretion onto a magnetized NS. As in the paper by Klein and Arons (1989), we do not postulate the existence of a shock in the column from the outset. Our study is unique in that we construct a hydrodynamic code based on the Godunov method, which allows us to deal with discontinuous flows and, in particular, to describe the shock dynamics.

The model consistently takes into account the kinetics of electron–ion beams in a strong magnetic field. The magnetic field in the column is assumed to be known, but the related processes of matter–radiation interaction play a crucial role in the evolution of the accretion flow.

In Section 1, we present our hydrodynamic model of the accretion flow. In Section 2, based on model profiles of the flow in the column, we consider the destruction probability of CNO nuclei in the accretion flow. The results obtained are discussed below.

1. ACCRETION FLOW HYDRODYNAMICS

Formulation of the Problem

We consider the time evolution of the accretion flow in the magnetic column above the polar cap of a magnetized NS at distances of no larger than several NS radii from its surface. We assume that the accretion is hydrodynamic, because the growth time of magnetohydrodynamic (MHD) instabilities is short under the conditions in question. Under the conditions of strong initial anisotropy of the accretion flow, the multistream instabilities can have characteristic growth rates comparable to the ion plasma frequency, $\omega_{\rm pi} \sim 1.2 \times 10^{12} n_{18}^{1/2}$ s^{-1}. Such growth rates are typical of the particle isotropization processes in collisionless shocks. Note that the cyclotron frequencies are much higher for typical magnetic fields. The characteristic propagation velocity of MHD disturbances is $V_{\rm A} \approx c(1-\alpha)$, where $\alpha = 10^{-11} n_{18} B_{12}^{-2} \ll 1$. This estimate can be obtained from general relations (see, e.g., the book by Velikovich and Liberman 1987). Since there are currently no microscopic simulations of instabilities under typical conditions of NS magnetic fields, we study the accretion flow by assuming that instability grows in a subrelativistic flow at a rate $\omega_{\rm pi}$. The simulations can be used to interpret the observations of X-ray pulsars.

The electrons and ions move in the accretion flow with the same mean (flow) velocity, but have different temperatures.

The NS is assumed to have a constant dipole magnetic field on the time scales under consideration. The geometry of the accretion column is shown in Fig. 1.

Basic Parameters and Equations

The basic model parameters include the mass ($M_\star$) and radius ($R_\star$) of the NS as well as the magnetic field strength ($B_\star$) at its magnetic pole and the accretion rate per unit area of the accretion column base ($\dot{M}/A_0$).

The system of hydrodynamic equations that describe the evolution of the flow may be written as

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathbf{u}) = 0, \quad (1)$$

$$\frac{\partial (\rho u_\alpha)}{\partial t} + \frac{\partial p}{\partial x_\alpha} + \frac{\partial}{\partial x_\beta}(\rho u_\alpha u_\beta) = \mathcal{F}_\alpha,$$

$$\frac{\partial}{\partial t}\left[\rho_s (E_s + \frac{u^2}{2})\right]$$

$$+\mathrm{div}\left[\rho_s \mathbf{u}(E_s + \frac{u^2}{2}) + p_s \mathbf{u}\right] = \mathcal{Q}_s,$$

where $\rho = \rho_e + \rho_i$, $p = p_i + p_e$, $\mathcal{F}$ and $\mathcal{Q}_s$ denote the momentum and energy sources, and $s = i, e$ is the type of particles.

We supplement this system with the equations of state for each type of particles. We use the equation of state for an ideal gas, $E_s = p_s/[\rho_s(\gamma_s - 1)]$. For

$x_s = k_\mathrm{B}T_s/(m_s c^2) \ll 1$, the adiabatic index γ_s may be written as $\gamma_s \approx \gamma_{0s}(1 - x_i)$ (de Groot *et al.* 1980), where $\gamma_{0i} = 5/3$ is a typical nonrelativistic value for particles with three degrees of freedom, and $\gamma_{0e} = 3$, since the electrons in the strong fields under consideration are quasi-one-dimensional.

The electron momentum distribution is one-dimensional, because the characteristic relaxation time of the electron Landau levels is $\sim 10^{-15} B_{12}^{-2}$ s, where $B_{12} = B/10^{12}$ G (see, e.g., Bussard 1980), and it is the shortest time in the system under consideration after the cyclotron time. The radiative decay time of the excited ion Landau levels is $\sim 5 \times 10^{-9} B_{12}^{-2}$ s, and is appreciably longer than the collisionless relaxation time of the ion momenta $\propto \omega_\mathrm{pi}^{-1}$. Therefore, in this case, the ions are at highly excited Landau levels and may thus be considered three-dimensional and be described quasi-classically.

We consider the *one-dimensional* motion of the accreting plasma along the *dipole lines* of a strong NS magnetic field. In this geometry, system (1) may be rewritten as

$$r^3 \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(r^3 \rho u) = 0, \tag{2}$$

$$r^3 \frac{\partial(\rho u)}{\partial t} + \frac{\partial}{\partial r}[r^3(p + \rho u^2)] = r^3 \mathcal{F} + 3r^2 p,$$

$$r^3 \frac{\partial}{\partial t}\left[\rho_s (E_s + \frac{u^2}{2})\right]$$

$$+ \frac{\partial}{\partial r}\left(r^3 \left[\rho_s u (E_s + \frac{u^2}{2}) + p_s u\right]\right) = r^3 \mathcal{Q}_s.$$

The system of equations (2) should be supplemented with initial and boundary conditions. As the initial condition, we consider a column filled with a cold, freely falling gas. The boundary condition in the upper part of the column is the condition for the inflow of a cold supersonic stream, while the boundary condition in the lower part of the column is the condition for the absence of a stream flowing into the star.

Physical Processes in the Accretion Flow

Here, we describe the processes that contribute to the terms $\mathcal{F}$ and $\mathcal{Q}_s$ of system (2).

Since we are considering a single-stream flow, the forces acting on the ions and electrons add up in one force term:

$$\mathcal{F} = F^i + F^e, \; F^i = F^i_\mathrm{grav} - F_\mathrm{atm},$$
$$F^e = F^e_\mathrm{grav} - F_\mathrm{nonres} - F_\mathrm{res},$$

where F^i_grav and F_atm denote the gravity and the viscous force of resistance to the flow (which is effective only in the NS atmosphere); and F^e_grav, F_nonres, and F_res denote the gravity and the nonresonant and resonant radiation pressure forces, respectively.

The gravitational force acting on a unit volume is

$$F_\mathrm{grav} = F^e_\mathrm{grav} + F^i_\mathrm{grav} = (nm_i + Znm_e)\frac{GM_\star}{r^2}, \tag{3}$$

where n is the ion density.

To calculate the nonresonant radiation pressure force, we use the following formula from the book by Zheleznyakov (1997):

$$F_\mathrm{nonres} = n_e \frac{\sigma_\mathrm{T}}{c} \frac{\sigma_\mathrm{ST} T_\gamma^4}{1 + \tau_\mathrm{T}}, \tag{4}$$

where n_e is the electron density, σ_ST is the Stefan–Boltzmann constant, T_γ is the local temperature of the radiation field, and τ_T is the nonresonant optical depth.

To calculate the resonant radiation pressure force, we numerically integrate the equation of radiative transfer in the cyclotron line, find the energy density of the photon field U_phot, and determine F_res as dU_phot/dr. Since the column in the cyclotron line is optically thick, the transfer equation may be written as the diffusion equation

$$\nabla \cdot \mathbf{J}_\mathrm{phot} = S_\mathrm{phot} + \frac{1}{3}\mathbf{u} \cdot \nabla U_\mathrm{phot}, \tag{5}$$

where S_phot are the cyclotron photon sources, and $\mathbf{J}_\mathrm{phot} = \frac{4}{3}\mathbf{u}U_\mathrm{phot} - \kappa \nabla U_\mathrm{phot}$ is the photon diffusion flux in the line. Since the diffusion of cyclotron photons across the magnetic field is severely hampered (see, e.g., Arons *et al.* 1987), we consider only the parallel component of Eq. (5), which may be written as

$$\frac{1}{r^3}\frac{\partial}{\partial r}\left\{r^3 \left[\frac{4}{3}U_\mathrm{phot} u - \kappa_{||}\frac{1}{r^3}\frac{\partial}{\partial r}\left(r^3 U_\mathrm{phot}\right)\right]\right\} \tag{6}$$
$$= S_\mathrm{phot} + \frac{1}{3}u\frac{1}{r^3}\frac{\partial}{\partial r}\left(r^3 U_\mathrm{phot}\right),$$

where $\kappa_{||}$ is the diffusion coefficient parallel to the magnetic field. This equation can be integrated by the shooting method. The boundary conditions for it are the following: the cyclotron photons freely escape from the upper boundary of the column, and their number on the stellar surface corresponds to a blackbody spectrum with temperature T_eff.

To calculate the friction force exerted on the flow from the NS atmosphere, we use the following standard expression for the Coulomb stopping in a dense environment:

$$F_\mathrm{atm} = \frac{4\pi n_\mathrm{a} n_i e^4 Z^2 \ln \Lambda}{m_e u^2}, \tag{7}$$

where Λ is the Coulomb logarithm, $n_{\rm a}$ is the electron density in the NS atmosphere, and u is the velocity of the accretion flow. A similar expression was used in the paper by Bildsten *et al.* (1992) devoted to the collisional destruction of CNO nuclei in the NS atmosphere.

The forces acting on the flow work on it, and this work is effectively redistribited between the ions and electrons.

Let the external force F^i act on the ions and the external force F^e act on the electrons. It then follows from local electrical neutrality (if the frequencies of the external-force variations are much lower than the characteristic plasma frequencies) that the fluxes of both types of particles are equal, i.e.,

$$(F^e - eE)\frac{n_e}{m_e\nu_{ei}} = (F^i + ZeE)\frac{n_i}{m_i\nu_{ei}},$$

where E is the ambipolar electric field, and ν_{ei} is the effective electron–ion relaxation frequency. Since $n_e = Zn_i$,

$$eE = F^e\frac{1}{\xi+1} - F^i\frac{\xi}{Z(\xi+1)},$$

where $\xi = m_e/m_i$, and the resulting effective force acting on the ions is

$$F^i_{\rm eff} = (F^i + F^e Z)\frac{1}{\xi+1},$$

while the resulting effective force acting on the electrons is

$$F^e_{\rm eff} = (F^i + F^e Z)\frac{\xi}{Z(\xi+1)}.$$

The ion energy changes through the following processes:

the small-angle scattering in Coulomb collisions with electrons H_{ie};

the collisional excitation of electron Landau levels $Q_{\rm cyc}$;

the collisional relaxation on electrons of the NS atmosphere $Q_{\rm relax}$;

the work done by the effective force, $F^i_{\rm eff}u$.

The cyclotron ion cooling was taken into account in the model. Its effect is noticeable if the dipole magnetic field exceeds 5×10^{11} G and if the system is transparent to photons in the proton cyclotron line.

The electron energy changes through the following processes:

the small-angle-scattering in Coulomb collisions with ions H_{ei};

the bremsstrahlung cooling in collisions with ions and electrons, Br_{ei} and Br_{ee};

the excitation of Landau levels in collisions with ions and electrons, Cyc_{ei} and Cyc_{ee};

the Compton processes $Q_{\rm compt}$;

the work done by the effective force $F^e_{\rm eff}u$.

For details on the calculations of these terms, see the Appendix.

The Method of Numerical Solution

A multicomponent accretion flow onto the NS surface can have discontinuities and, in particular, shocks. For this reason, we chose the standard Godunov method (Godunov 1959; Godunov *et al.* 1976) to solve the problem.

The Godunov method is directly applicable to one-component[1] systems that are described by equations without any sources. To include the sources in Eqs. (2) in the numerical scheme, we use an approach based on the works by Le Veque (1997). It consists in splitting system (2) into two parts: one part describes the conservation of the fluxes along dipole lines and can be integrated using a modified Godunov method, and the other part describes the presence of energy and momentum sources and can be integrated by the Gear method.

The necessity of using this approach stems from the multicomponent nature of the system, which contains two types of particles that interact with each other, with the external magnetic and gravitational fields, and with the radiation field in a complicated way.

We make system (2) dimensionless by multiplying $\mathcal{F}$ by $C_F = \frac{t_\star}{\rho_\star u_\star}$ and $\mathcal{Q}_s$ by $C_Q = \frac{t_\star}{\rho_\star u_\star^2}$, where $t_\star$, $u_\star$, and $\rho_\star$ are the characteristic time, velocity, and density scales, respectively.

Equations (2) in the column are integrated by a combined method that allows the Godunov scheme to be generalized to systems with energy and momentum exchange between its components (the sources in system (2)). We break down the accretion column into spatial cells centered on points $x_{i-1/2}$. From the initial state at time $t=0$ to the state at the current time t, the integration is performed over the steps Δt in each of which the following operations are performed:

(1) The integration of the equations without sources corresponding to the conservation of the fluxes along dipole lines:

$$r^3\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial r}(r^3\rho u) = 0, \tag{8}$$

[1] The currently available multitemperature methods were described, for example, by Zabrodin and Prokopov (1998).

$$r^3\frac{\partial(\rho u)}{\partial t}+\frac{\partial}{\partial r}[r^3(p+\rho u^2)]=0,$$

$$r^3\frac{\partial}{\partial t}\left[\rho_s(E_s+\frac{u^2}{2})\right]$$
$$+\frac{\partial}{\partial r}\left(r^3\left[\rho_s u\left(E_s+\frac{u^2}{2}\right)+p_s u\right]\right)=0.$$

At this stage, we integrate the system simultaneously in the entire column, i.e., in all cells $x_{i-1/2}$.

(2) The integration of the equations with the momentum and energy sources written for the quantities averaged over the spatial cell:

$$\frac{\partial(\rho_i u_i)}{\partial t}=C_F\mathcal{F}_i+\frac{3}{r_i}p_i, \tag{9}$$
$$\frac{\partial}{\partial t}\left[\rho_{s_i}(E_{s_i}+\frac{u_i^2}{2})\right]=C_Q\mathcal{Q}_{s_i},$$

where q_i are the values of the physical quantities averaged over cell i, independently in each of the cells $x_{i-1/2}$. At each of the two stages, the integration is performed over the same time interval specified by the Courant condition at the first stage.

Since system (9) is stiff, we use the standard LSODE subroutine (Hindmarsh 1983), in which version B of the Gear method (see, e.g., Gear 1971) is implemented, for its integration. Note that many popular methods for integrating stiff systems that are based on the modified Bulirsch–Stoer method (Press *et al.* 1983) require an explicit specification of the Jacobian of the system. In our case, this leads to significant complications, because system (9) has a complex structure of its right-hand parts that includes the terms specified only in numerical form.

To integrate system (8), we used the capacitive modification of the Godunov method of the first order suggested by LeVeque (1997). The essence of this method is that, if the conservation law for the physical quantity $q(x,t)$ is written in generalized form instead of the classical divergence form,

$$\kappa(x)\frac{\partial q(x,t)}{\partial t}+\frac{\partial f(q(x,t))}{\partial x}=0,$$

where $\kappa(x)$ is a given function of the spatial coordinate that denotes the effective capacity of the region of space under consideration (e.g., the porosity of the medium), then the conventional Godunov difference scheme is inapplicable, and instead of the Godunov standard expression for the grid function q at time $t_0+\Delta t$

$$\tilde{q}_i=q_i-\frac{\Delta t}{\Delta x_i}\left(F_i-F_{i+1}\right),$$

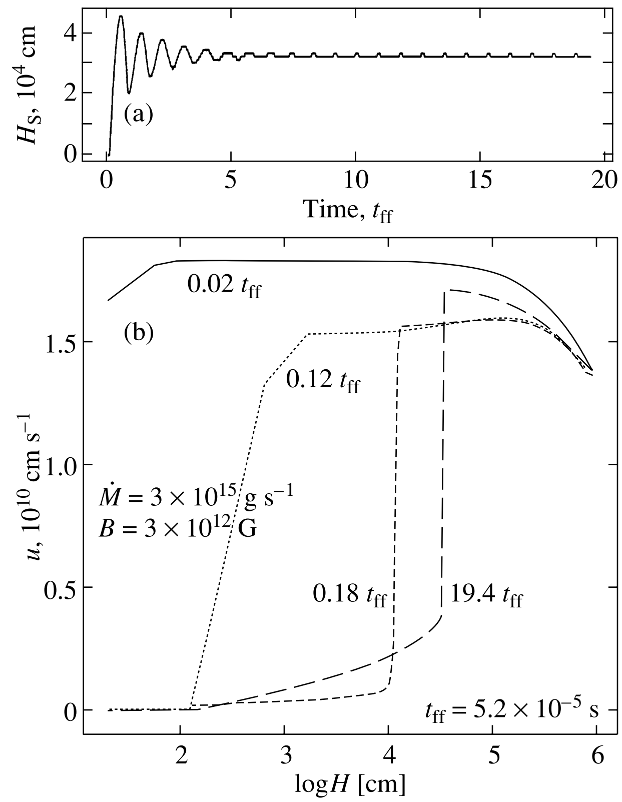

Fig. 2. Evolution of the accretion flow: (a) shock height versus time; (b) evolution of the flow velocity profile.

where F_i is the q flux flowing from cell i into cell $i-1$, we may write

$$\tilde{q}_i=q_i-\frac{\Delta t}{\kappa_i\Delta x_i}\left(F_i-F_{i+1}\right),$$

where Δx_i is the size of cell i, and κ_i is the value of $\kappa(x)$ averaged over cell i.

Results

We have implemented the complex scheme of computations[2] described above and investigated the evolution of the accretion flow for various model parameters.

We have found that strong shocks develop in the column on time scales of $\sim10^{-5}$ s. These shocks execute stable oscillations about their equilibrium positions with periods of $\sim10^{-5}$ s that are damped out in a time of $\sim10^{-3}$ s. A typical example of the evolution of the shock front and the flow velocity profile is shown in Fig. 2.

[2]Some of the computations were carried out under the support of the St. Petersburg Branch of the Interdepartmental Supercomputer Center (http://scc.ioffe.ru/).

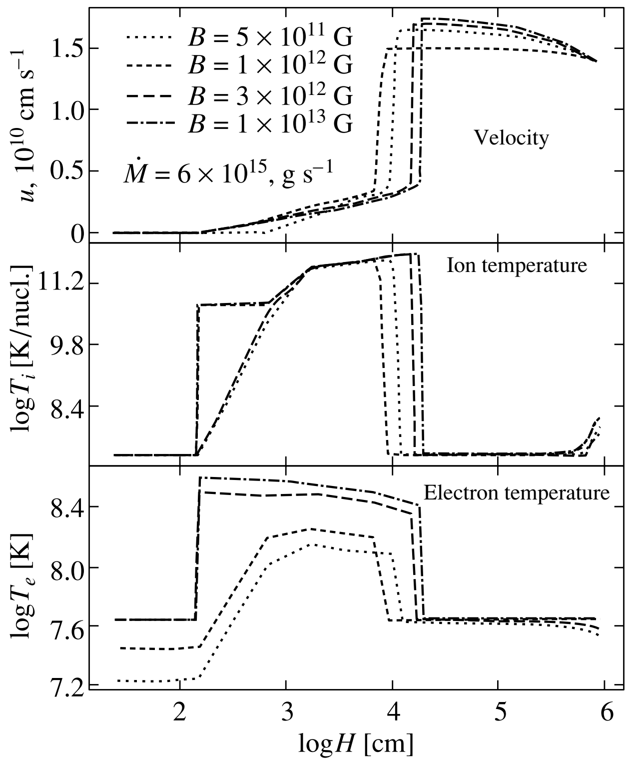

Fig. 3. Dependence of the flow profiles at $\tau = 1.3 \times 10^{-3}$ s on the magnetic field strength.

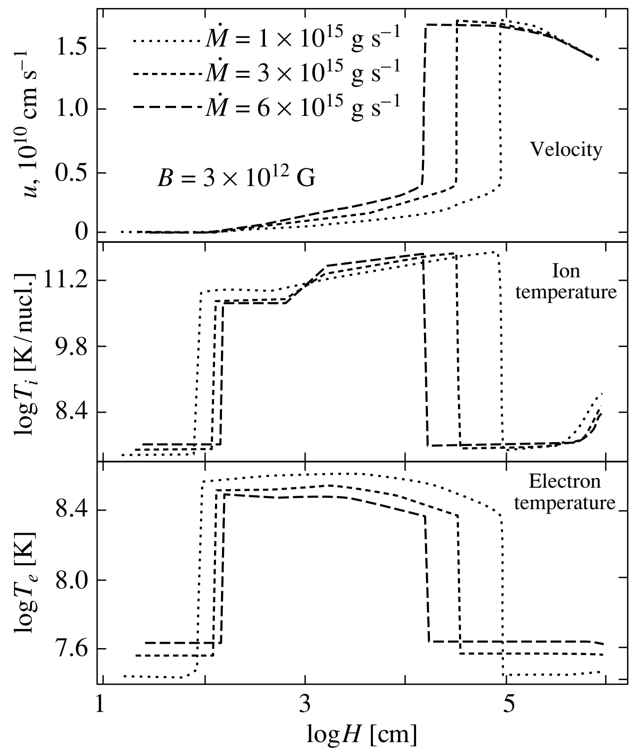

Fig. 4. Dependence of the flow profiles at $\tau = 5 \times 10^{-4}$ s on the accretion rate.

The model flow profiles shown in Figs. 3 and 4 exhibit stable and strong shocks that decelerate and heat the accretion flow. The ions at such shocks heat up much more strongly than the electrons, because they contain the bulk of the flow kinetic energy. However, as the heated ions move further toward the surface, they give up much of their energy to the electrons, which, in turn, release their energy in the form of cyclotron and bremstrahlung photons and give it up to the nonresonant photons in Compton collisions.

In most cases, the compression ratio at the shock fronts slightly exceeds 4 (the maximum value for nonrelativistic single-fluid shocks) because of the mildly relativistic changes in the adiabatic index of the ions heated to several tens of MeV.

An important property of the model is the transformation of a substantial fraction of the flow energy into the energy of photons in an optically thick electron cyclotron line. This fraction is plotted against the NS magnetic field strength in Fig. 5. The pressure of the trapped cyclotron radiation is dynamically significant for the deceleration of the accretion flow. The cyclotron ion cooling is significant for magnetic fields stronger than 5×10^{11} G. In this case, the accretion regime depends significantly on the structural features of the magnetic fields in the NS atmosphere at heights of less than 10^3 cm from the surface. At these heights, the field can differ greatly from a dipole field due to the local fields of higher multipolarity. A nonuniform field structure makes the column transparent to optical and X-ray photons of the proton and electron cyclotron radiation. Figure 6 shows the accretion regimes in the optically thin case where the ion cyclotron radiation freely escapes from the column. Allowing for the cyclotron ion cooling leads to the shock front approaching the NS surface if the dipole magnetic field exceeds 5×10^{11} G. In this case, the radiation spectrum of the system exhibits a prominent optical/ultraviolet proton cyclotron line. Because of the strong deceleration and effective cooling of the flow in the column in the optically thin case, only about half of the accretion flow energy reaches the stellar surface (see Fig. 7).

If the magnetic field has a regular structure, the proton cyclotron line can become optically thick at heights of $\sim 10^3$ cm from the NS surface. In this case, the accretion regimes are similar to those in Fig. 3, since the collisionless relaxation with a high frequency restores the isotropy of the ion distribution function faster than does the radiative change in the transverse (with respect to the field) temperature.

It should be noted that the difference in the shapes of the flow profiles at low and high magnetic field strengths (Fig. 3) stems from the fact that the electron temperature downstream of the shock front is

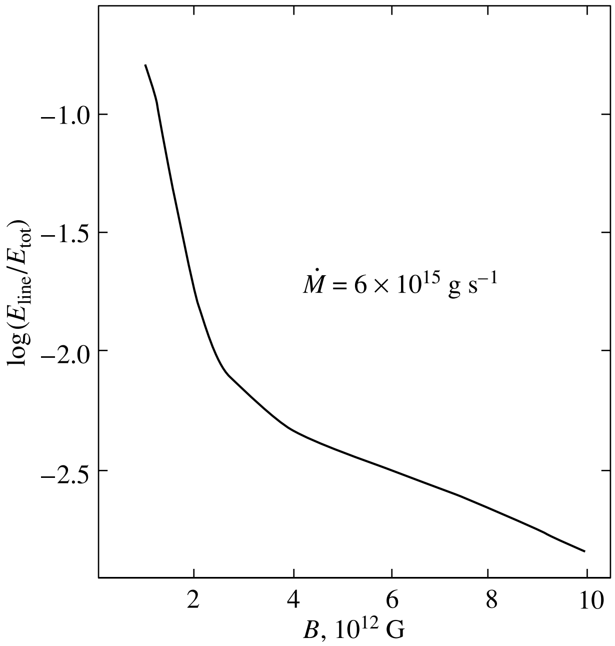

Fig. 5. Logarithm of the fraction of the flow free-fall energy converted into the energy of cyclotron radiation versus magnetic field strength.

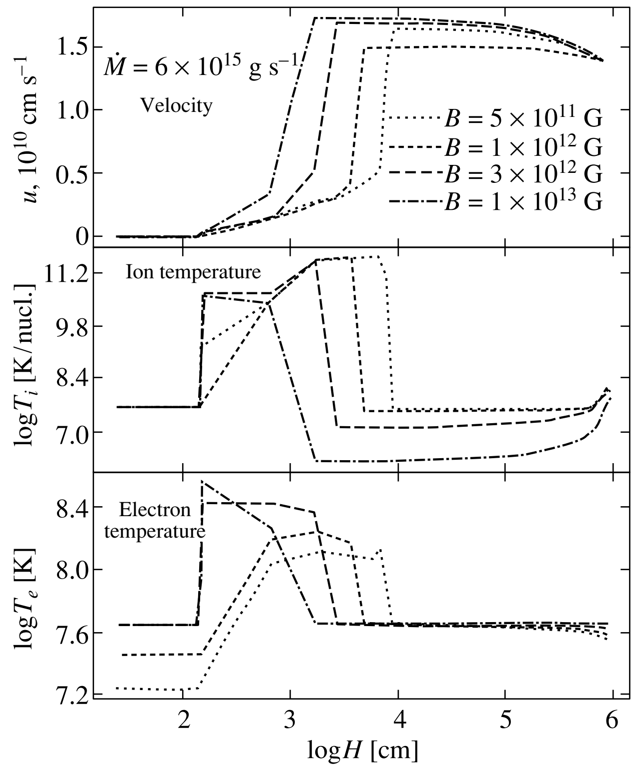

Fig. 6. Dependence of the flow profiles at $\tau = 0.8 \times 10^{-3}$ s on the magnetic field strength for the case where the column is optically thin for the cyclotron photons emitted by ions.

lower for weak fields. Therefore, the gradient in the energy density of the photons trapped in the cyclotron line increases toward the surface, providing a radiative pressure that contributes to a more effective flow deceleration.

2. THE DESTRUCTION OF CNO NUCLEI IN THE ACCRETION COLUMN

The chemical composition of the material that reaches the NS surface is an important question in the theory of accretion flows. This question is important, in particular, for the theory of X-ray bursts, because the CNO nuclei are the catalyst for the thermonuclear hydrogen burning reaction on the NS surface (see Lewin *et al.* (1993) and references therein). It is well known that the spallation reactions of nuclei with energies of several tens of MeV per nucleon or higher in the accretion flows of compact sources may lead to the destruction of nuclei and to a significant decrease in the gamma-ray fluxes in lines (see, e.g., Aharonian and Sunyaev 1984; Bildsten *et al.* 1992). This question was considered by Bildsten *et al.* (1992) for the case where the accretion flow decelerates in a dense NS atmosphere through Coulomb losses. These authors conclude that almost all of the CNO nuclei will be destroyed before they reach the NS surface. However, they note that this conclusion may be invalid in cases where the flow decelerates in the column above the atmosphere.

There is a fundamental possibility of collisionless flow deceleration through collective plasma effects and effective energy removal by electron radiation in a magnetic field. In this case, the thickness of the material traversed by a nucleus as it decelerates to energies of $\sim$10 MeV per nucleon can be appreciably smaller than the thickness traversed by a nucleus in the case of its purely Coulomb deceleration to the same energy. Having constructed the model density, velocity, and ion temperature profiles in the column, we can answer the question of how effective the destruction of CNO nuclei in the accretion flow is and where they are destroyed.

To quantitatively verify this effect, we calculated the destruction probability of a carbon nucleus that was accreting together with the flow (since the nitrogen and oxygen destruction cross sections are close to the carbon destruction cross section, the destruction of these nuclei will be similar).

We numerically integrated the cross section for the destruction of carbon nuclei by protons determined by Read and Viola (1984) and obtained the frequencies of the destroying collisions per unit volume. We then calculated the dependences of the optical depths with respect to destruction and the destruction probabilities on the distance to the NS surface.

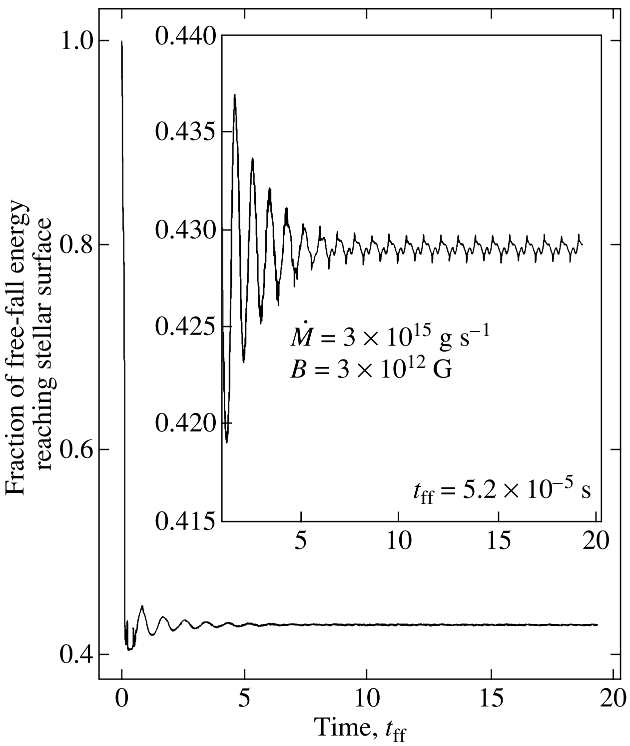

Fig. 7. Time evolution of the fraction of the flow energy that reaches the stellar surface.

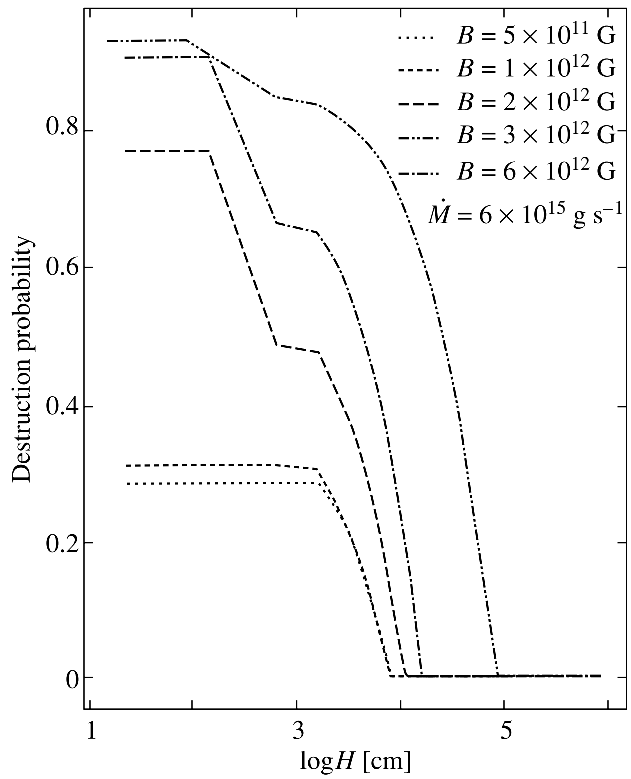

Fig. 8. Destruction probability of a ^{12}C nucleus versus distance to the stellar surface.

The destruction probability of a carbon nucleus is plotted against the distance to the NS surface in Fig. 8 for a set of magnetic field strengths at the NS pole. It follows from this figure that, for moderately strong magnetic fields, a significant fraction of the nuclei can reach the stellar surface and, hence, can be the catalyst for X-ray bursts.

CONCLUSIONS

We have constructed a numeral model for the unsteady accretion of material in a one-dimensional column above the polar region of a magnetized NS. The model gives a two-fluid description of the plasma accretion flow in a strong dipole magnetic field. The model is unique in that it uses the Godunov method for numerically integrating the flows of matter with discontinuities in the form of shocks. Using this method, we have been able to reveal and investigate the time evolution of the shocks in the plasma accretion flow. A steady flow with an accretion shock is established after a period on the order of several plasma free-fall times. The ion temperature abruptly increases at the shock front to $\sim 10^{11}$ K, which weakly depends on the magnetic field strength and the accretion rate. Depending on the magnetic field strength, the electron temperature reaches $(3–5) \times 10^8$ K.

Part of the accretion flow energy transforms into the energy of cyclotron radiation in an optically thick line whose pressure affects significantly the plasma flow deceleration. In this case, a substantial fraction of the flow kinetic energy is released in the form of optically thin radiation without reaching the bottom of the column. It is commonly assumed that the kinetic energy of the flow and the radiation from the optically thin part of the column that reach the bottom of the column transform into blackbody radiation in the optically thick region of the NS atmosphere. The radiation of the accretion flow energy in the optically thin part of the column leads to a tangible decrease in the effective temperature of the blackbody radiation from the polar spot $T_{\rm eff}$. In general, the *a priori* dependence $T_{\rm eff} \propto \dot{M}^{1/4}$ breaks down, since the fraction of the energy radiated in the optically thin part of the column is a complex function of $\dot{M}$.

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APPENDIX

THE RATES OF PARTICLE–FIELD ENERGY EXCHANGE

(1) We use the cooling rates in the case of small-angle scattering H_{ei} (and $H_{ie} = -H_{ei}$) from Langer and Rappoport (1982):

$$H_{ei} = 2\sqrt{\frac{2}{\pi}} r_0 n_e n \xi \frac{T_i - T_e}{T_e + \xi T_i} Z^2 \sqrt{\frac{m_e c^2}{k_{\rm B}(T_e + \xi T_i)}} \Lambda, \quad (A.1)$$

where Λ is the Coulomb logarithm, and $r_0 = 4\pi r_e^2 m_e c^3$ is the characteristic energy loss scale in this process.

(2) To calculate the cooling rates $Q_{\rm cyc}$ and Cyc_{ei}, we numerically integrated the total quantum-electrodynamic cross section for the collisional excitation of electron Landau levels in a strong magnetic field from Langer (1981). To obtain an acceptable result over wide ranges of temperatures and magnetic field strengths, we took into account the excitation of the first ten Landau levels in the integration.

(3) To calculate Cyc_{ee}, we used a fitting formula from Langer and Rappoport (1982):

$$Cyc_{ee} = 2.04 r_0 n_e^2 B_{12}^{-1/2} \sqrt{\frac{k_{\rm B} T_e}{\hbar\omega_B}} \times \exp\left\{ -\frac{m_e c^2}{k_{\rm B} T_e}\left(\sqrt{1 + 0.04531 B_{12}} - 1\right)\right\} \times \left(\frac{B_{12}}{5}\right)^{\left(\frac{k_{\rm B}T_e}{9597\ {\rm keV}}\right)^{0.2}}. \quad (A.2)$$

(4) To calculate the bremsstrahlung cooling rate Br_{ee}, we numerically integrated the cross section from Haug (1975) and derived the fitting formula

$$Br_{ee} \approx 2.5410 \times 10^{-37} T_e^{1.45811} n_e^2 g(B, T_e), \quad (A.3)$$

where $g(B, T_e) = (0.409 - 0.0193 B_{12} - 0.00244 B_{12}^2) \times (k_{\rm B} T_e / 10\ {\rm keV})^{0.25}$ is the Gaunt factor from Langer and Rappoport (1982).

(5) To calculate the bremsstrahlung cooling rate Br_{ei}, we numerically integrated the cross section from the book by Berestetskii *et al.* (1980) for high electron temperatures and used the fit from Langer and Rappoport (1982) for low electron temperatures:

$$Br_{ei} \approx \begin{cases} 0.36\alpha r_0 (T_e/T_e^b)^{0.5} n_e n_i Z^2 g(B, T_e), & T_e < T_e^b \\ 0.36\alpha r_0 (T_e/T_e^b)^{1.2} n_e n_i Z^2 g(B, T_e), & T_e \geq T_e^b, \end{cases} \quad (A.4)$$

where $T_e^b = 5 \times 10^8$ K.

(6) $Q_{\rm relax}$ denotes the Coulomb relaxation of the accretion flow on electrons of the thin and dense NS atmosphere. We used the simple model of an isothermal atmosphere and determined $Q_{\rm relax}$ as

$$Q_{\rm relax} = -\nu_{ei} \frac{k_{\rm B} n_i}{\gamma_i - 1}(T_i - T_{\rm atm}), \quad (A.5)$$

where ν_{ei} is the Coulomb collision frequency.

(7) $Q_{\rm compt}$ denotes the electron cooling in single Compton scatterings. For moderately hard photons ($\frac{\gamma E_\gamma}{m_e c^2} \ll 1$) that scatter nonrelativistic electrons, the energy lost by an electron in a single nonresonant scattering is $\Delta E = -\frac{E_\gamma^2}{m_e c^2} + \frac{4 k_{\rm B} T_e E_\gamma}{m_e c^2}$. Since $\langle E_\gamma \rangle = 3 k_{\rm B} T_\gamma$, $\langle E_\gamma^2 \rangle = 12 k_{\rm B} T_\gamma^2$,

$$Q_{\rm compt} = n_e n_\gamma \langle \sigma_{\rm T} v_{\rm rel} \Delta E \rangle H(B, T_\gamma) = 12 n_e n_\gamma \sigma_{\rm T} c k_{\rm B} T_\gamma k_{\rm B} \frac{T_e - T_\gamma}{m_e c^2} H(B, T_\gamma), \quad (A.6)$$

where $H(B, T_\gamma) = (1 + 0.0165(\hbar\omega_{\rm B}/k_{\rm B}T_\gamma)^{2.48})/(1 + 0.0825(\hbar\omega_{\rm B}/k_{\rm B}T_\gamma)^{2.48})$ is the Gaunt factor for the process under consideration from Arons *et al.* (1987); and n_γ and T_γ are the local density and temperature of the photon field, respectively.

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